

## MATHEMATICAL MODELING OF AN ADAPTIVE ACOUSTO-OPTIC INTERFERENCE CANCELLER

K. B. Kostin

e-mail: konst@pop.ioffe.rssi.ru

Baltic State Technical University, 1<sup>st</sup> Krasnoarmeyskaya str. 1, 198005 St. Petersburg, RUSSIA

## ABSTRACT

The analysis of operation of novel adaptive acousto-optic interference canceller is conducted. Based on the analysis, the mathematical model and algorithm of operation of the canceller is developed.

Keywords:

- optical adaptive processing;
- acousto-optics;
- interference cancellation;
- prediction.

## INTRODUCTION

At present optico-electronic adaptive processors are widely used in different areas of modern technique and technology - Refs1, 2. In Ref.3 a novel scheme of adaptive acousto-optic processor for wide-band interference cancellation was proposed. In the proposed processor the transmission-type liquid crystal light valve (LCLV) is applied as the time integrator and the acousto-optic cell (AOC) is used on one hand for operating as a delay line and on the other hand for multiplying the error signal with delayed samples of reference noise. This scheme is of a great interest when multi-channel optical adaptive processors (OAP) operating with wide-band signals are designed since the number of acousto-optic modulators is reduced from two to one and the alignment of the processor is significantly simplified - Ref.3. In the present paper, the operation of the proposed processor is analyzed. Based on the analysis, the mathematical model and algorithm of operation of the processor is developed.

## MODELING OF OAP

The OAP for interference cancellation under consideration represents the closed feedback loop processor. The task of the processor is to predict the

current value of noise  $\hat{n}(t)$  (its estimation) via the known reference noise  $n_r(t)$  on the base of its former value linear combination.  $\{n_r(t-\tau), \tau \in (0, T]\}$ . The predicted value of noise is:

$$\hat{n}(t) = \int_0^T n_r(t-\tau) w(\tau, t) d\tau \quad (1)$$

T-observation time,  $w(\tau)$ -weight coefficients, that provide the minimum root-mean-square error between the current and predicted values of noise (when Least Mean Square (LMS) adaptation algorithm is used, which is realized in the interference canceller considered here).

The difference signal therewith  $e(t) = n(t) - \hat{n}(t)$  represents the estimation error. The optimal weight coefficients are determined as follows -Ref.4:

$$w(\tau, t) = G \int_{-\infty}^{\infty} e(\xi) n_r(\xi - \tau) h(t - \xi) d\xi \quad (2)$$

$\tau \in (0, T]$ .

One can see from expression (2) that optimal weight coefficients represent the mutual correlation function of the difference signal  $e(\xi)$  and received noise  $n_r(\xi - \tau)$ . The function  $h(t)$  is used as a weight. On the other hand, the expression (2) can be considered as a convolution of the product of  $e(\xi) n_r(\xi - \tau)$  and the function  $h(t)$ . Hence the optimal weight coefficients represent the result of product  $e(\xi) n_r(\xi - \tau)$  transmission through the filter with impulse response  $Gh(t)$ . Here  $G$  is a constant that determines the adaptation speed (amplification coefficient in the closed feedback loop).

Thus, the main operations performed with the signals in a linear predictor are the following: the signal delay in time, the signal multiplication, their correlation and integration. These particular operations are easily realized in coherent and acousto-optics.

The scheme of the OAP for noise cancellation is presented in Fig. 1. OAP has two inputs.

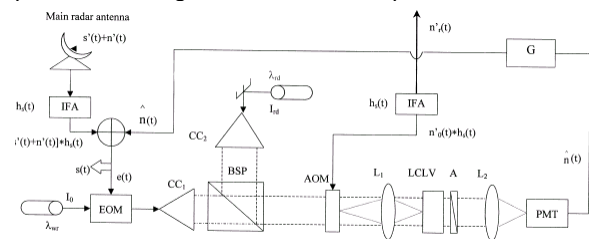


Fig.1. Optico-adaptive noise canceller

The main input is acted upon the additive mixture of signal and noise  $s'(t) + n'(t)$  from the main radar antenna. The reference noise (interference)  $n_r(t)$  from the auxiliary omnidirectional antenna is applied to the second (auxiliary) input. The signal processing is performed at the intermediate frequency after frequency conversion. The additive mixture of signal and noise from the output of the intermediate frequency amplifier (IFA) is applied to the input of the summator (differential amplifier). The predicted value

of noise  $\hat{n}(t)$  in negative polarity is applied to the other input of the summator. The error signal from

the summator output  $e(t) = s(t) + n(t) - \hat{n}(t)$  is applied to the input of optico-adaptive noise canceller. The OAP main elements are: acousto-optic modulator (AOM), used as a delay line and transmission-type liquid crystal light valve (LCLV), which serves as time integrator.

Let us consider the operation of OAP for noise

cancellation in more detail. The writing beam laser intensity is modulated in time by the error signal in the electro-optical modulator (EOM).

Due to the fact that modulation of EM has  $\sin^2$  form and, therefore, nonlinear, in order to provide linear regime of its work the voltage bias is introduced. Therefore, the light intensity at the output of EOM is modulated with the error signal  $e(t)$  that can be expressed under first-order approximation as:

$$I_{out} = [0.5 + \alpha \cdot e(t)] I_0 \eta \quad (3)$$

where  $I_0$  is laser writing light intensity;  $\eta$  – EOM transmission and  $\alpha$  is the slope of the modulation function of EOM. The output beam, passing through collimator CC<sub>1</sub> and beam splitter (BSP), illuminates acousto-optic (AO) modulator AOM in a form of stripe controlled by auxiliary channel noise  $n_r(t)$ . The collimator introduces the aperture function  $a(x/v)$  describing non-uniformity of the optical beam (Gaussian distribution).

Thus optical wave at the output of AOM has intensity distribution:

$$I_{out} = I_0 \eta [0.5 + \alpha \cdot e(t)] \cdot [0.5 + \beta \cdot n_r(t - x/v)] \rho(x/v) a(x/v) \quad (4)$$

where  $\beta$  is the modulation function slope of AOC, and  $\rho(x/v)$  describes the attenuation of the acoustic wave along its propagation within the crystal of AOM.  $v$  is the speed of the acoustic wave propagation. Therefore we have obtained the intensity distribution along the  $x$  coordinate (along the cell aperture), which contains the product of the error signal  $e(t)$  and the delayed samples of reference noise  $n_r(t - x/v)$ , namely:

$$0.25 + 0.5\beta n_r(t - x/v) + 0.5\alpha e(t) + \alpha\beta e(t) n_r(t - x/v) \quad (5)$$

Only one of the terms of (5) contains product of error signal  $e(t)$  and the delayed samples of reference noise  $n_r(t - \tau)$  which is now to be integrated over time to obtain their mutual correlation function  $e(t) * n_r(t - \tau)$ . Also, all other noninformative terms in (5) must be removed. If both  $e(t)$  and  $n_r(t - \tau)$  do not contain frequency components at frequencies below several hundred hertz, which is almost always the case, contribution of these non-informative terms can be ignored. In order to realize the computation of the mutual correlation function  $e(t) * n_r(t - \tau)$  the transmission-type liquid crystal light valve (LCLV), which is an optically controlled spatial light modulator, manufactured by S.I. Vavilov State Optical Institute, Russia is used. The output light from AOM goes to spherical lens L1 that forms image on the writing side of the LCLV. Its performance is based on rotation of polarization plane of linearly polarized reading beam according to the spatial intensity distribution of the writing beam. Angle of polarization plane rotation at a given point  $x$  depends on both intensity of the writing light wave  $I(x, t)$  and time  $t$ :

$$\theta(x, t) = \int_{-\infty}^{\infty} I(x, \xi) h(t - \xi) d\xi \quad (6)$$

where  $h(t)$  is the impulse response of the LCLV. For LCLV this response  $h(t)$  normally has the following form:

$$h(t) = \begin{cases} 0 & \dots \dots \dots t < 0 \\ \exp(-t / \tau_{LC}) & \dots t \geq 0 \end{cases} \quad (7)$$

where  $\tau_{LC}$  is the modulator time constant that is of the order of tens of milliseconds. Thus, the reading light

plane of polarization is rotated with the following time-spatial distribution of angles of rotation:

$$\begin{aligned} \theta(x, t) &= I_0 a(x/v) p(x/v) \eta \times \\ &\times \int_{-\infty}^{\infty} [0.25 + 0.5\alpha e(\xi) + 0.5\beta n_r(\xi - \frac{x}{v}) + \alpha\beta e(\xi) n_r(\xi - \frac{x}{v})] h(t - \xi) d\xi \approx \\ &\approx \frac{I_0}{4} a(x/v) \rho(x/v) \eta \int_{-\infty}^{\infty} h(t - \xi) d\xi + \\ &+ \alpha\beta I_0 a(x/v) \rho(x/v) \eta \int_{-\infty}^{\infty} e(\xi) \cdot n_r(\xi - \frac{x}{v}) \cdot h(t - \xi) d\xi \end{aligned} \quad (8)$$

Since there are not many devices that are capable of operating with polarization modulated light, polarization modulation must be converted to intensity modulation. This conversion is done by using analyzer A. Transmission function of analyzer has form of  $\sin^2[\theta(x) - \theta_0]$  where  $\theta_0$  is the angle between polarization plane of reading light and analyzer axis. In order to achieve linear response for small signals, DC bias is selected to be  $\theta_0 = 45^\circ$  which is achieved by proper selection of parameters in (8) so that the first term is equal to  $\pi/4$ . Thus, the writing light intensity distribution at the output of the analyzer is:

$$0.5 + \alpha\beta \eta \beta a(x/v) \rho(x/v) \int_0^{\tau_{LC}} e(\xi) \cdot n_r(\xi - \frac{x}{v}) \cdot h(t - \xi) d\xi \quad (9)$$

We have obtained the weighted mutual correlation functions of the error signal and reference noise, i.e. the distribution of the weight coefficients.

The next and basically final step on the way of

obtaining current value of main channel noise  $\hat{n}(t)$  is to produce convolution operation of the reference noise samples with the weight coefficients. In order to do that the reading light laser beam is used. The reading light with the intensity  $I_{rd}$  from the second laser is applied through CC2 (identical to CC1) and BSP to the AOM, where  $I_{rd}$  is modulated with the delayed samples of noise  $n_r(t - \tau)$ . As a result, the following intensity distribution is present at the AOM output:

$$I_{out}(x, t) = [0.5 + \beta \cdot n_r(t - x/v)] a(x/v) \rho(x/v) I_{rd} \quad (10)$$

By means of lens L<sub>1</sub> this distribution is projected on the light sensitive surface of the LCLV. As a result, the intensity distribution in the transmitted through LCLV reading light wave will be the product of (9) and (10), i.e. the multiplication of the calculated weight coefficients  $w(t, x)$  and the delayed noise samples  $n_r(t - \tau)$  is being done on the reading light wavelength in the LCLV itself:

$$\begin{aligned} &[0.5 + \alpha\beta \eta \beta a(x/v) \rho(x/v) \int_0^{\tau_{LC}} e(\xi) n_r(\xi - \frac{x}{v}) h(t - \xi) d\xi] \times \\ &\times [0.5 + \beta n_r(t - \frac{x}{v})] a(x/v) \rho(x/v) I_{rd} \end{aligned} \quad (11)$$

In order to integrate over the spatial coordinate  $x$  the lens L<sub>2</sub> is used. It focuses this distribution as a spot on the photo-sensitive surface of the photo multiplying tube (PMT). As a result, the current at the PMT output will be proportional to the integral from the expression (11), integrating by  $dx$ :

$$\begin{aligned} I_{PMT} &= \varepsilon \int_D [0.5 + \alpha\beta \eta \beta a(x/v) \rho(x/v) \int_0^{\tau_{LC}} e(\xi) n_r(\xi - \frac{x}{v}) h(t - \xi) d\xi] \times \\ &\times I_{rd} a(x/v) \rho(x/v) [0.5 + \beta n_r(t - \frac{x}{v})] dx \end{aligned} \quad (12)$$

suppresses all signal components with frequencies less than 30-500 Hz and neither  $e(t)$  nor  $n_r(t)$  contain frequency components of less than  $1/\tau_a$  ( $\tau_a$ -time aperture of the AOM) expression (12) can be written as follows:

$$i_{PMT} = \alpha \beta^2 I_0 I_{rd} \eta \int_0^D [a^2(x/v) \rho^2(x/v) n_r(t - \frac{x}{v}) \times \int_0^{\tau_{ic}} e(\xi) n_r(\xi - \frac{x}{v}) h(t - \xi) d\xi] dx \quad (13)$$

As a result we obtain the expression for the predicted noise, taking into account that  $\tau = x/v$ :

$$\hat{n}(t) = GA \int_0^{\tau_a} a^2(\tau) \rho^2(\tau) n_r(t - \tau) \times \int_0^{\tau_{ic}} e(\xi) n_r(\xi - \tau) h(t - \xi) d\xi d\tau \quad (14)$$

where  $G = K R_l$  – constant;

$K$  – amplification coefficient of the amplifier in the feedback loop;

$R_l$  – the resistance of the PMT load;

$$A = \alpha \beta^2 I_0 I_{rd} \eta.$$

If we assume that  $a(\tau) = 1$  and  $p(\tau) = 1$  and (i.e. neglect the influence of the collimator and AOM aperture functions), it is obvious that expression (14) is completely identical to (1). On the base of analysis performed, the operation algorithm of the optical adaptive noise canceller is presented on Fig.2.

noise

## CONCLUSIONS

On the base of the novel processor operation analysis the mathematical model and operation algorithm are designed. This gives a possibility to investigate the physical behavior of the processor under different external interference conditions and to estimate the influence of the AOC and LCLV characteristics on the interference cancellation ratio.

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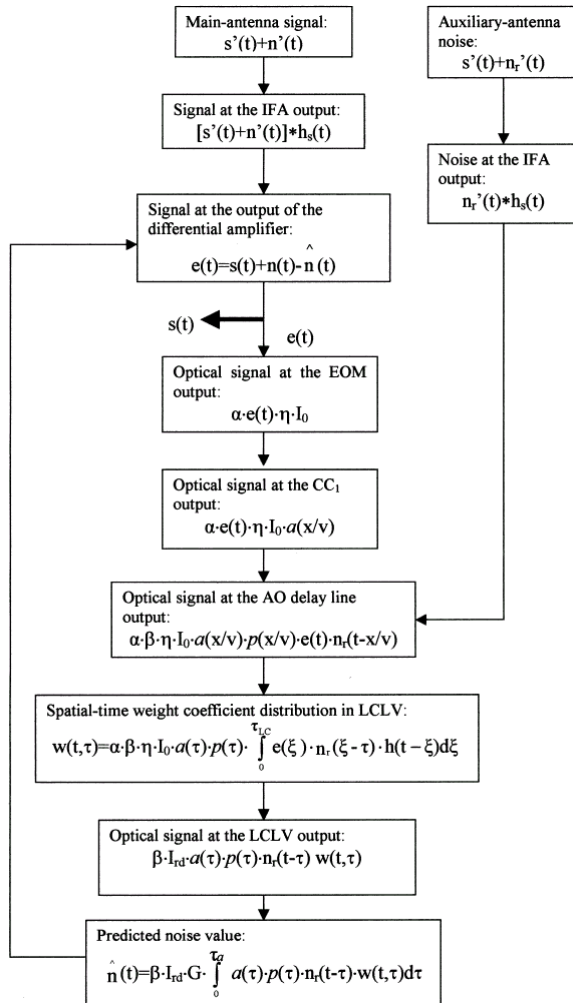


Fig. 2. Operation algorithm of the optical adaptive canceller.